647510 **BSCabsCC6020** **Seat No :________**

B.Sc. Semester - 6 (*CBCS*) Examination
May/June-2021 (OLD COURSE)
MATHEMATICAL ANALYSIS-2 AND ABSTRACT ALGEBRA -2(CORE)

Time: 1:30 Hours **Marks: 42**

Instructions:

1. Figures to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Q.1(A) **Answer any one:** (07)

(1) Prove that compact subset of a metric space is closed and bounded.

(2) State and prove theorem of nested intervals.

Q.1(B) **Answer any one:** (04)

(1) Define: Connected subset of metric space. Prove that every singleton subset of any metric space is connected set.

(2) Give examples of two disjoint sets which are not separated and prove your claim.

Q.1(C) **Answer any one:** (03)

(1) If F is a closed subset of metric space X and K is a compact Subset of metric space X then prove that $F \cap K$ is compact.

(2) Give an example of a subset of a metric space R which is neither compact nor connected.

Q.2(A) **Answer any one:** (07)

(1) Define: Laplace transform. Prove that

(i) $L(cosat) = \frac{s}{s^2+a^2}$ (ii) $L(sinhat) = \frac{a}{s^2-a^2}; s > |a|$.

(iii) $L(t^n) = \frac{n!}{s^{n+1}}; n = 0,1,2, \dots ..$

(2) Define: Inverse Laplace transform.

Find: $(i) L^{-1}\left\{\frac{2s^2-4}{(s+1)(s-2)(s-3)}\right\}$ $(ii) L^{-1}\left\{\frac{s+1}{s^2(s^2+1)}\right\}$

Q.2(B) **Answer any one:** (04)

(1) Find the Laplace transform of

(i) $f(t) = t, 0 < t < 4$
$\quad = 5, t > 4$

$(ii) f(t) = t, 0 \leq t \leq 2$
$\quad = 2, t > 2$

(2) Find: (i) $L(sin^3 2t)$ (ii) $L^{-1}\left\{\frac{2s+6}{s^2+4}\right\}$

Q.2(C) **Answer any one:** (03)

(1) State and prove first shifting theorem for Laplace transformations.

(2) Find (i) $L^{-1}\left(\frac{1}{s^3}\right)$ (ii) $L(e^t cos2t)$

Q.3(A) **Answer any one:** (07)

(1) Derive the formula for finding Laplace transform of the n^{th} derivative of $f(t)$. Using it find the Laplace transform of (i) sinat (ii) t^n

(2) State convolution theorem and using it find

(i) $L^{-1}\left\{\frac{s^2-a^2}{(s^2+a^2)^2}\right\}$ (ii) $L^{-1}\left\{\frac{1}{(s-1)(s^2+1)}\right\}$.

Q.3(B) **Answer any one:** (04)

(1) If $L\{f(t)\} = \bar{f}(s)$ then prove that $L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}\left[\bar{f}(s)\right]$, where n = 1,2,3….

(2) Evaluate: (i) $L\left(\frac{cos2t-cos3t}{t}\right)$ (ii) $L\frac{(sint)}{t}$

Q.3(C) **Answer any one:** (03)

(1) Evaluate: (i) $L\left(\sqrt{t}e^{2t}\right)$ (ii) $L(tcosht)$

(2) Evaluate: (i) $L^{-1}\left[\frac{1}{s(s-1)}\right]$ (ii) $L^{-1}\left[cot^{-1}\left(\frac{s}{a}\right)\right]$

Q.4(A) **Answer any one:** (07)

(1) Define: Kernel of homomorphism. Prove that: A homorphism $\varphi:(G,*) \to (\mathrm{G}', \Delta)$ is one-one iff $K_{\varphi} = \{e\}$.

(2) Prove that the commutative ring R with unity is a field if it has no proper ideal.

Q.4(B) **Answer any one:** (04)

(1) If I_1 and I_2 are ideals of ring R then prove that $I_1 \cup I_2$ is also an ideal of R iff either $I_1 \subset I_2$ or $I_2 \subset I_1$.

(2) If R is a ring then show that $U = \{a \in R: ab = ba, \forall b \in R\}$ is subring of R.

Q.4(C) **Answer any one:** (03)

(1) Define: (i) Unit element in a ring (ii) Left and right ideal (iii) Principal ideal ring.

(2) If $G = (\mathrm{R}, +)$and $\varphi: G \to G, \varphi(x) = x + 1, \forall x \in G$ then show that φ is not a homomorphism.

Q.5(A) **Answer any one:** (07)

(1) State and prove division algorithm for polynomials.

(2) If $f(x), g(x) \in F[x] - \{0\}$ then show that there exists their g.c.d. $d(x)$ and $d(x) = a(x)f(x) + b(x)g(x)$, for some polynomials $a(x), b(x)$ of $F[x]$.

Q.5(B) **Answer any one:** (04)

(1) State and prove remainder theorem for polynomials.

(2) Factorize $f(x) = x^4 + 4 \in Z_5[x]$ by using factor theorem.

Q.5(C) **Answer any one:** (03)

(1) Define: Quaternion number and simplify: (i) $(1 - 2i + 2k)^2$ (ii) $(2i - 3j)^2$

(2) Define: Inverse of quaternion. Simplify:

(i) $[(3 + 2i)(4j + 5kj)^{-1}]$ (ii) $(1 + i + k)^{-1}(1 - j + k)$

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